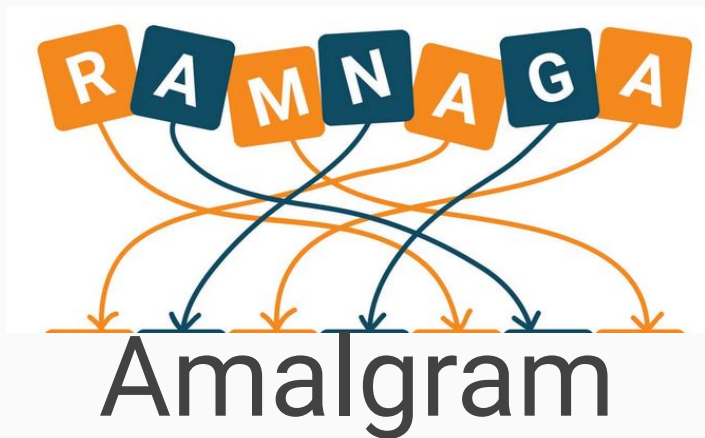


UKIEPC 2024



Summary and solution outlines

Problem Solutions



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Overview

- Create a word containing:
 - All of the letters of word1
 - All of the letters of word2
- If a letter occurs:
 - N times in word1
 - M times in word2
 - Then it should occur at least $\max(N,M)$ times in word3

Amalgram

Techniques

- Hashmaps
- Sorting

Algorithm

- We need a way to take the superset of both strings
- One method inspired by merge-sort:
 - Sort both of the strings
 - Create two pointers i, j . For as long as $i < n, j < m$:
 - If $a[i] == b[j]$: Print $a[i]$ and advance **both** $i++$, $j++$
 - Otherwise if $a[i] < b[j]$: Print $a[i]$ and advance $i++$
 - Otherwise: print $b[j]$ and advance $j++$
- Other methods exist
 - Like counting frequency in a hashmap for each string



Budget Analysis

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Overview

- You are given a formula for linear regression and must optimise the parameters to the formula
- The optimisation takes place on many different subarrays of the same list
- Invent a way to do this efficiently (faster than $O(N)$ in the size of the list)

Budget Analysis

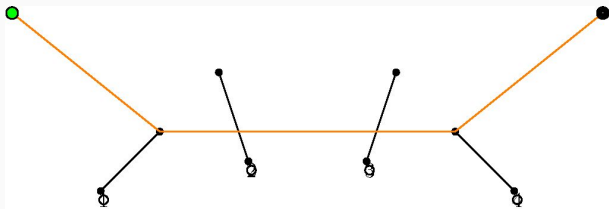
Techniques

- Calculus
- Prefix sums

Algorithm

- Taking the partial derivatives and solve the resulting linear equations system.
- One could see that for the linear regression of y on x (even if it is regularized) one needs to know the following:
 - $\sum(x_i)$, $\sum(y_i)$, $\sum(x_i^2)$ and $\sum(x_i * y_i)$.
- Given that the given x_i and y_i are constant, we could precompute the prefix sums and get the regression coefficients for each query in $O(1)$. Computing the prediction also takes constant time.

Overall time complexity of the solution is $O(n+m)$



Cross Country

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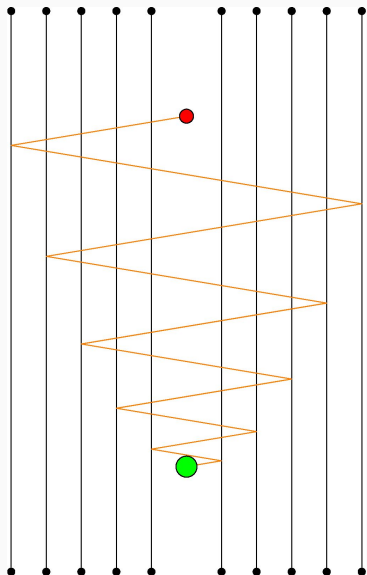
Overview

- Draw the shortest line going through some line segments A, B, C, D, ... Z
- Going through a line segment multiple times is OK, but you must go through the segment in the right order too.

Cross Country

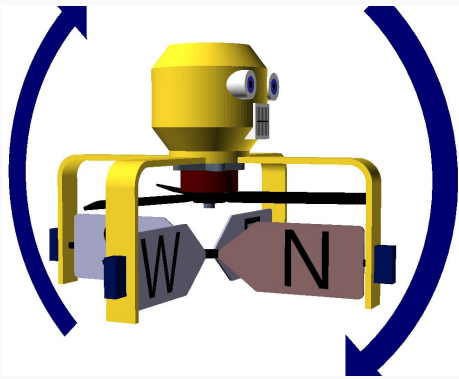
Techniques

- Geometry
- Dynamic programming



Algorithm

- When we encounter a line segment on our path, there are three potentially-optimal moves:
 - Keep going in the same direction
 - Keep going in a **reflection** of the same direction
 - Completely change direction
- The shortest path will only **completely** change direction if it touches the endpoint of a line
 - Otherwise the path could be shortened by changing angles
- Eliminate the reflection aspect by generating 2^N scenarios of “virtual line segments” where reflection is already simulated.
- Then it’s a shortest path problem:
 - Two segment endpoints have a connecting edge **iff** the line between them goes through all intermediate segments.



Drone Operator

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Overview

- Given is a system of 3 equations in 4 variables "n", "e", "s", "w"
 - This system is underspecified, but relates all of the variables to each other.
- The objective is to make the maximum of $|n|$, $|e|$, $|s|$, $|w|$ as small as possible under the constraints.

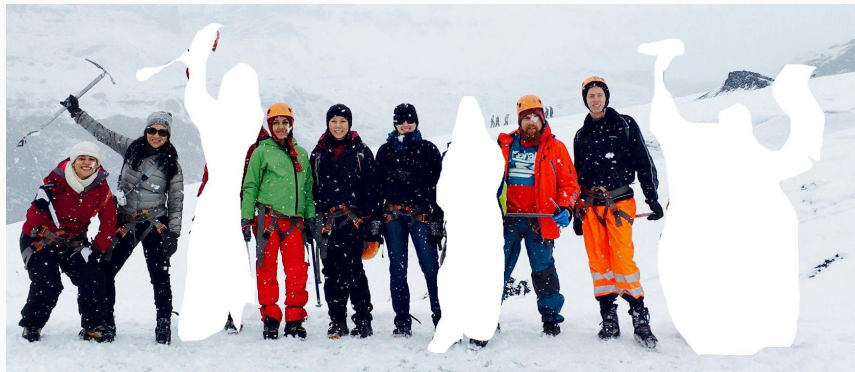
Drone Operator

Techniques

- Greedy algorithms
- Sorting

Algorithm

- Our equations have one spare degree of freedom:
 - $p = e - w \quad \quad \quad :::: \quad w = e - p$
 - $r = n - s \quad \quad \quad :::: \quad s = n - r$
 - $y = n + e + s + w \quad \quad :::: \quad y = 2n + 2e - r - p$
- In a solution where $\max(|n|, |e|, |s|, |w|) = T$, **two** variables have $|v| = T$.
 - Otherwise, $\max(|v|)$ could be decreased without increasing T .
 - The options are: ($|n|=|e|$, $|n|=|s|$, $|n|=|w|$, $|e|=|s|$, $|e|=|w|$)
- So either 1. Add each possible equation into the system and solve algebraically, taking the minimum solution as the answer.
- Or 2. Binary search on the value of T , and for every pair in (n, e, s, w) set both elements to T then check if this gives a valid set of equations.



Eradication Sort

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Overview

- Remove some items from an unsorted list, to make it sorted
- For each gap of width W you leave in the resulting list, you pay a cost of W^2
 - What is the minimum total cost you can achieve?

Eradication Sort

Techniques

- Dynamic programming
- Binary indexed trees
- Convex line functions

Algorithm

- Straightforward dynamic programming gives a (too slow) solution:
 - $\text{cost}[x]$ = the cost in case the last item **kept** so far is $y[x]$
 - $\text{cost}[x] = \min(\text{cost}[i] + (x-i)^2 \text{ for-all } [i < x] \text{ st. } y[i] \leq y[x])$
 - Complexity of implementation is $O(N^2)$
- We need ways to find the candidate values, and to get the minimum among them, quickly.
 - $\text{cost}[x] = \text{cost}[i] + (x-i)^2$ can be rearranged as a line equation:
 - $\text{cost}[x] - x^2 = \text{cost}[i] + i^2 - (2i)x$
 - If we ignore $(y[i] \leq y[x])$, we can store the lines in a list sorted by gradient, and use binary/two-pointers search to find the best line for a given x .
- To address the $(y[i] \leq y[x])$ constraint, we'll create a binary-indexed tree of these lists. After calculating $\text{cost}[x]$, update $\text{lines}[y[x] \dots \text{inf}]$.



Finding Proteins

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Overview

- The coordinates of some points with high dimensionality are given
- We will build a subset of these points and call it K.
 - K is built by repeatedly taking the point which is **farthest** from the **closest** point in K
- Find a way to build the set K quickly.

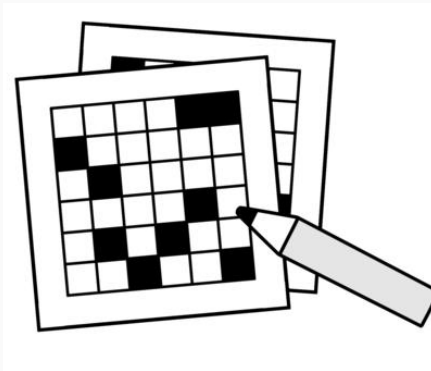
Finding Suspicious Proteins

Techniques

- Ad-hoc optimisation
- Precalculation

Algorithm

- When choosing the next value V (which happens K times):
 - We need to calculate $\min(\text{dist}(V, k[i]) \text{ for all existing } k$
 - This is $(O(L \cdot K))$ - too slow
 - However, $\min(\text{dist}(V, k[0 \dots i]))$ is the same as:
 - $\min(\text{dist}(V, k[0 \dots i]), \text{dist}(V, k[i+1]))$
 - The only thing that changes is the addition of the new element of K , and this can be updated later.
 - We can maintain the current minimum distances to so-far-unselected items as $\text{dist}[0 \dots n-1]$
 - Every time we choose a new element of K , update all the distances with $\min(\text{cur}, \text{dist}(k))$



Grid Search

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Overview

- You are given a 2D word and a 2D grid in which to search for that word
 - Highlight the places where the word occurs in the grid.

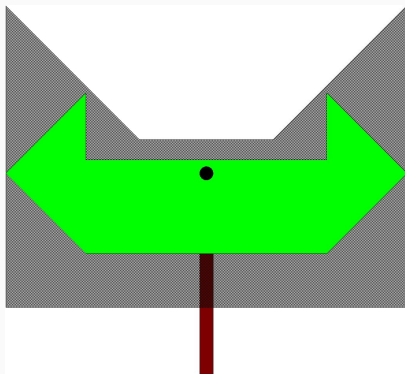
Grid Search

Techniques

- Hashing
- FFT

Algorithm

- String-matching algorithms based on hashing (Rabin-Karp) and FFT translate well over to multiple dimensions
 - For example, using a polynomial hash we can use 2 passes:
 - For each row, save the running hash of the last N items
 - Then for each column, make a running hash of the last M hashes (using P^N or some other prime for rows)
 - Knowing the size of the word in advance helps with this, eg. building prefix sum tables for each row.
- Standard string-matching with Aho-Corasick is adaptable enough to work too.
 - First, for each row find which string(s) match at each column
 - Then, look in each column for the right pattern of match IDs
 - Both steps are one-dimensional string matching problems.



Hedge Topiary

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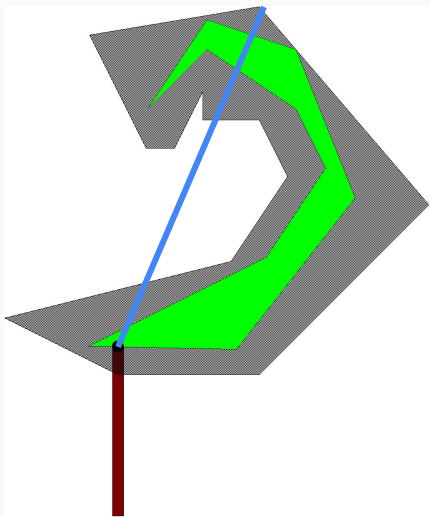
Overview

- You have two shapes, A and B.
 - The shapes may be convex.
 - However, the shapes are simple polygons with integer-aligned vertices.
- Shape A needs to fit inside B.
 - How big can you scale A (around the origin) so that it fits inside B?

Hedge Topiary

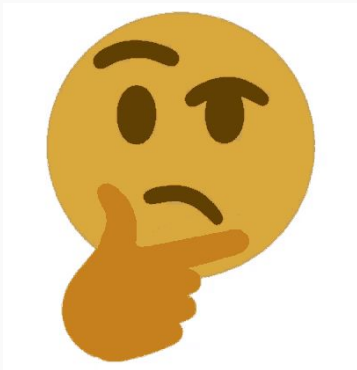
Techniques

- Ray-ray intersection
- Sweepline
- Rational arithmetic



Algorithm

- Imagine slowly scaling shape A up. At times, the boundaries of A cross inside/outside the boundaries of B (and vice versa)
 - Such intersections happen at **vertex/edge boundaries** (a vertex of A/B touches an edge of B/A).
- We're going to solve the problem individually for a "radial slice" in the direction of each vertex of A+B and combine solutions later.
 - For each vertex in the input, find the one-dimensional "slices" of A and "slices" of B in the direction of that vertex.
 - We need to align these slices so that they overlap.
 - For each slice attached to a vertex, find the set of feasible ranges of scale factors where it can "fit" into/around the slices of the other shape
- Finally, use a sweepline to intersect the sets of feasible regions.



Inconsistent

0 correct • not solved

Overview

- We want to demonstrate Simpson's Paradox.
- Two teams have been training equally hard across N categories of problem
 - Team1 solved $a[i]/b[i]$ of category i
 - Team2 solved $c[i]/d[i]$ of category i
- Both teams attempted M problems in total.
- Generate a table of results where team1 has done better than team2 on each individual category, but team2 has solved more tasks overall.

Inconsistent Patterns

Techniques

- Construction

Algorithm

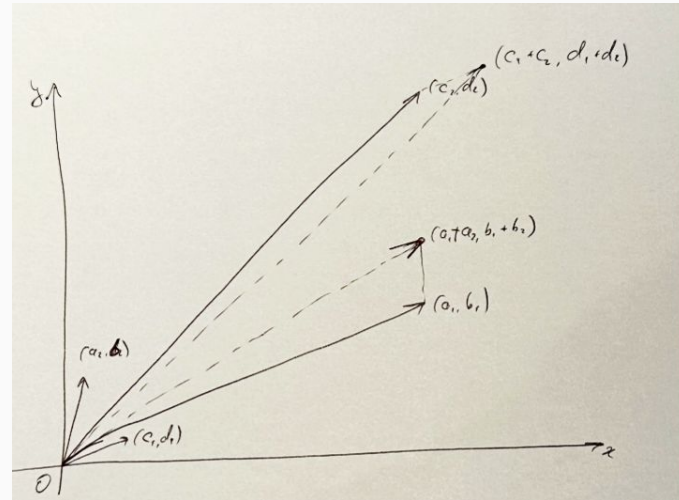
- Given the $M \geq 4N$ one construction that works is as follows:
 - $(M - 3 \cdot (N - 1) - 1) / (M - 3 \cdot (N - 1)) < 1/3$
 - $1/3 < 1/2$: repeated $(N - 2)$ times
 - $1/3 < x / (M - 1 - 2 \cdot (N - 2))$
 - where x is the smallest integer such that this inequality holds, ie $\text{floor}(M - 1 - 2 \cdot (N - 2))$

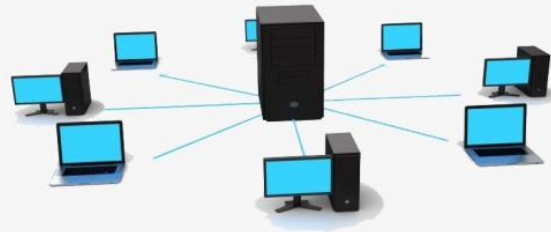
The intuition behind this construction is looking at (a_i, b_i) and (c_i, d_i) as two-dimensional vectors.

We need pairs of vectors such that:

- the slope of vector1 is greater than the slope of vector2
- while this order reverses when we add up all first elements is pairs and all second

($\text{sum}(a_i) / \text{sum}(b_i)$) correspond to the vector sum of (a_i, b_i) .





Jabber Network

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Overview

- You start with a connected, unrooted binary tree of computers (nodes) and connections (edges)
- Distances between nodes are defined using “demands”
- Periodically an edge will be removed from the tree, and you need to re-connect the graph in the cheapest way possible according to demands $\times$ distance.

Jabber Network

Techniques

- DFS on trees
- Precomputation

Algorithm

- Once an old link is removed, the graph is split into two connected components, and you need to reconnect them again.
- You can search for the endpoints of a new edge independently in each of these 2 components
- ...

Jabber Network

Techniques

- DFS on trees
- Precomputation

Algorithm

- Our way of searching for an endpoint in a component:
 - Identify all demands (s_i, t_i, d_i) that leave that component
 - i. (s_i) being a vertex in the current component
 - The contribution of the endpoint p is the sum of products of demand d_i times the distance between s_i to p
 - This contribution can be recomputed efficiently when you move p to one of its neighbours
 - to do that, precompute sums of demand products in each subtree using a DFS
 - This makes it possible to find the best endpoint using another DFS
- Complexity: $O(n(n + d))$: n iterations, each iteration takes $O(n + d)$



Knitting

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Overview

- Complete the design of a scarf made out of patchwork stripes
 - Within a range of K stripes, the same colour shouldn't occur more than once
 - Otherwise, make sure that no colour appears too often (minimise the frequency of the most-frequent colour)

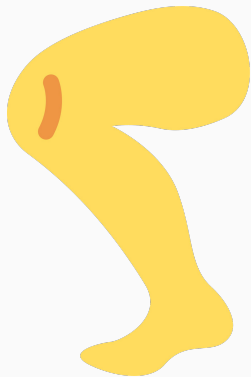
Knitting

Techniques

- Greedy algorithms
- Priority queues

Algorithm

- If a scarf can be constructed, we can build it with a greedy algorithm
 - Go from left-to-right filling in spaces.
 - Scan the last $K-1$ items of the existing scarf, looking for colours that we can't use for now.
 - Put the remaining colours in a priority queue where the **least-used** colour is on top (it's always best to use this if available)
 - Every time we add a colour from the priority queue, remove it from the queue and add scarf[i-k+1] to the queue instead.
- Make sure to check the resulting scarf is valid! (And that the priority queue never ran out of items)



Leg Day

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Overview

- Given a pattern of up to 31 strings, represent those strings using emojis and repeat those emojis up to 31 times to make a nice calendar
- The emojis you pick should match the strings, eg. "todayislegday" should be represented by a "🦵"

Leg Day

Techniques

- Unicode
- Grit

Algorithm

- Make sure to print the week numbers at the start of each line!
- For each string, use your standard library's string-in-string functions to check which rule it matches first (**in the order specified in the input**)
- C++ and Python have no problem outputting UTF-8 characters in a non-UTF8 context. When printing in Java, make sure to set the encoding on Strings properly.

Final Standings

<http://ukiepc2024.cloudcontest.org/>

